
Game Theoretic Modeling of WiFi pricing.

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Overview

- Motivation
- Results Preview
- General Formulation
- Web Browsing Model
- File Transfer Model
 - Bounded Length
 - Unbounded Length
- Bayesian Model
 - Fixed Length
 - Unbounded Length
- Ongoing Work

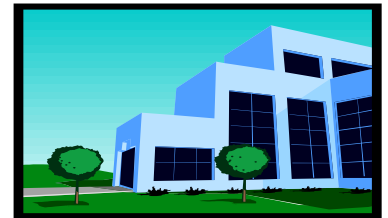
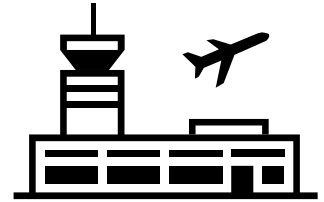
Motivation

■ Path to Universal WiFi Access

- ❑ Massive Deployment of 802.11 base stations for private LANs
- ❑ Payment scheme might incentivize base station owners to allow public access.

■ Direct Payments

- ❑ Avoid third party involvement.
- ❑ Transactions need to be "self enforcing"



Motivation



Client



Base Station

■ Payment Model

- Want Little or no third party involvement.
- Pre-pay? → Base station might take the money and run.
- Post-pay? → Client might take the service and not pay.
- Pay as you go?

■ Will this payment model work?

- Will the base station charge a fixed price over session duration?
- Will client and base station accept this payment model at all?

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Results Preview

- **Web Browsing Model**
 - Base Station charges a constant price.
 - Clients with high enough utilities connect.
- **File Transfer Model: Bounded File length**
 - Clients are "pessimistic" and refuse to pay anything until the last time slot.
 - Base station price not constant.
- **File Transfer Model: Unbounded File Length**
 - Base station price not constant.
- **Bayesian, "mixed" model, Fixed file length**
 - Base station charges fixed price if client is unlikely to be a file transferor.

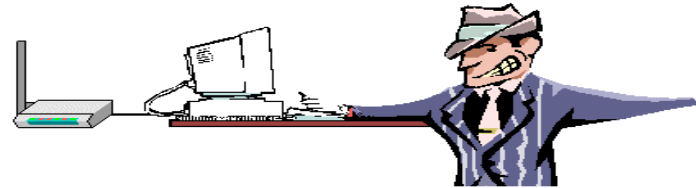
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General Formulation



Client



Base Station

■ Two Player Game

- Client
- Base Station

■ Each time slot, Base station proposes price, p_i

- If accepted, client pays & connects during slot i
- If rejected, client leaves.

■ If client accepts in slot i ,

- Base station proposes a new price p_{i+1} for slot $i+1$

General Formulation

- Client's utility at game's end: $f(t, K)$
 - K : intended session length
 - *max # slots client would like to connect.*
 - t : number of slots actually connected
 - Client's net payoff:

$$f(t, K) - \sum_{i=1}^t p_i$$

- Base Station payoff:

$$\sum_{i=1}^t p_i$$

Overview

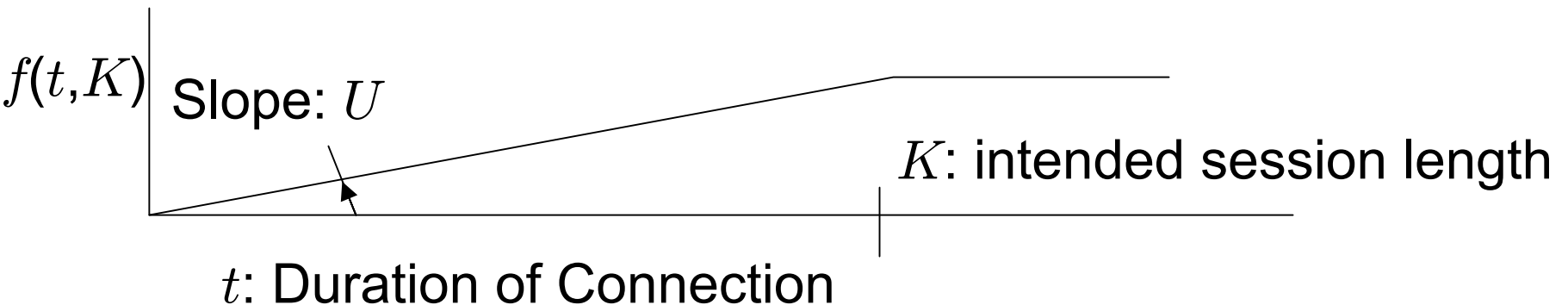
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Web Browsing Model

- Client's utility proportional to actual time connected, up until intended session length.

$$f(t, K) = U \min(t, K)$$

- **Asymmetric information:**
 - Base Station knows a distribution of (U, K) .
 - Client knows the sample value of (U, K) .



Web Browsing Model

Theorem 1

■ Assume:

- U, K , independent, finite mean
- The function $pP(U \geq p)$ has a unique maximizer: p^*

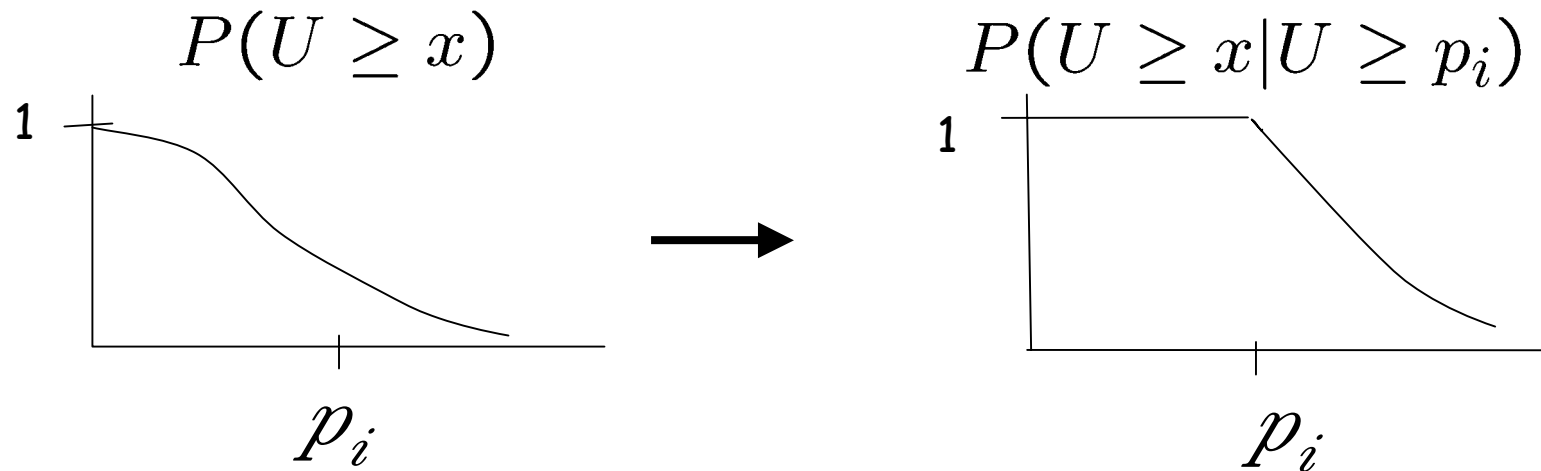
■ Then: The following strategy profile is a Perfect Bayesian Equilibrium (PBE):

- The Client connects or remains connected in slot i iff
 - $p_i \leq U$ and $i \leq K$ (the "myopic strategy")
- The Base Station charges the price p^* in all time slots.

Web Browsing Model

■ Why is fixed price surprising?

- When a myopic client accepts price p_i , the base station refines his conditional distribution of U .



- One might think that the base station would want to raise the price after learning this.

Web Browsing Model

■ Proof of Theorem 1

- Suppose the Client follows a myopic strategy.
- The Base Station
 - Should never decrease price.
 - Choose from non decreasing sequences to maximize:

$$\max_{\{p_i\}_{i=1..∞} \in P^+} \left[\sum_{i=1}^{\infty} p_i P(p_i \leq U) P(K \geq i) \right]$$

- Maximized when $p_i = p^*$ for
 - recall, p^* is the maximizer of $pP(U \geq p)$

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File Transfer Model

- Client's utility depends on being connected the entire intended session length.

$$f(t, k) = \begin{cases} 0 & \text{if } t < k \\ k & \text{if } t = k \end{cases}$$

- **Asymmetric Information**
 - Base Station knows distribution of K
 - Client knows sample value of K
 - No " U " parameter

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File Transfer: Bounded Length

Theorem 2

- If one assumes:
 - K distributed on $\{1, \dots, N\}$
 - $P(K=i|i)$ has a unique maximizer \hat{i}
- Then : The unique Nash equilibrium is the following strategy profile:
 - Client follows the "Pessimistic Strategy:"
 - Connects in slots prior to K iff $p_i = 0$
 - Connects in slot K , iff $p_i \leq K$
 - Base Station charges

$$p_i = \begin{cases} 0 & \text{if } i < \hat{i} \\ i & \text{otherwise} \end{cases}$$

File Transfer: Bounded Length

Proof of Theorem 2:

- Backwards Induction
- Iterated Deletion of Strictly Dominated Strategies
- The “pessimistic strategy” is dominant for clients of type N .
 - Suppose the game reaches time slot N
 - Client's dominant strategy is to accept any price $\leq N$
 - Base station's dominant strategy is to charge N .
 - A type N client would not accept non zero prices in slots prior to N .

File Transfer: Bounded Length

Proof of Theorem 2 (cont'd)

- Clients of type $N-1$ play the “pessimistic strategy”
 - Suppose not, and a type $N-1$ client pays a nonzero amount in a slot prior to $N-1$
 - When the game reaches time $N-1$, the base station knows that type N clients would have already quit, so the client must be type $N-1$.
 - The base station exploits the client and charges $N-1$
 - Thus it would have made no sense for the type $N-1$ Client to pay in slots prior to $N-1$.
 - → Type $N-1$ clients play the “pessimistic strategy.”

File Transfer: Bounded Length

Proof of Theorem 2 (cont'd)

- Clients of all types play the "pessimistic strategy."
 - Induction
- A base station facing "pessimistic" clients expect to have only one chance to charge a nonzero price.
 - Finds \hat{i} to maximize $P(K=i)i$
 - Charges

$$p_i = \begin{cases} 0 & \text{if } i < \hat{i} \\ i & \text{otherwise} \end{cases}$$

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File Transfer: Unbounded Length

- If there were always some probability that the game will continue, maybe the outcome would be different.
- Finite horizon case depended heavily on having a fixed point from which to work back.
- Infinitely repeated Prisoner's dilemma very different from finitely repeated version.



File Transfer: Unbounded Length

Theorem 3

■ Assume:

- The client is a File Transferor.
- K distributed on $\{1, 2, \dots\}$
- $P(K = i \mid K \geq i) > \delta$ for some $\delta > 0$
- $E[K \mid K \geq i]$ finite
- (Example: geometric)

■ Then:

- Constant price is not a Perfect Bayesian Equilibrium.

File Transfer: Unbounded Length

Proof of Theorem 3:

Consider...

- The "constant price" strategy:
 - Charge x in every slot, for some $x: 0 < x \leq 1$.
- The "alternative" strategy for the continuation game after some slot \hat{i}
 - Charge \hat{i} in slot \hat{i} .

File Transfer: Unbounded Length

Proof of Theorem 3 (cont'd):

- The base station should favor the “alternative” strategy in the continuation game if:

$$\hat{i}P(k = \hat{i} | k \geq \hat{i}) > xE[k | k \geq \hat{i}]$$

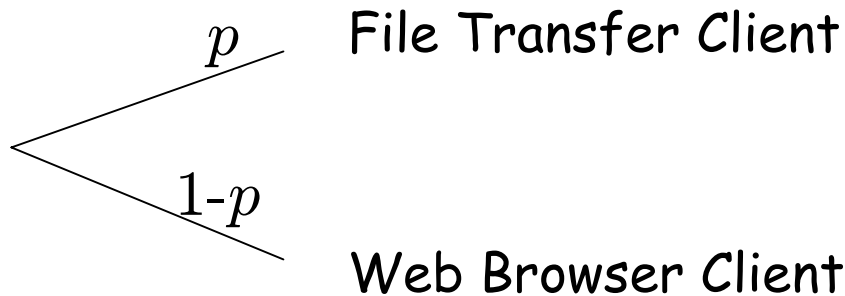
- There is always an \hat{i} large enough for the above inequality to hold.

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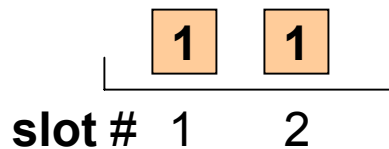
Bayesian Game: Fixed Length

- Assume

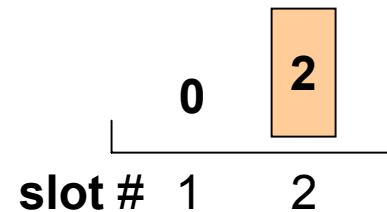


- Intended session length, K , fixed at 2.

- A WB client earns



- A FT client Earns



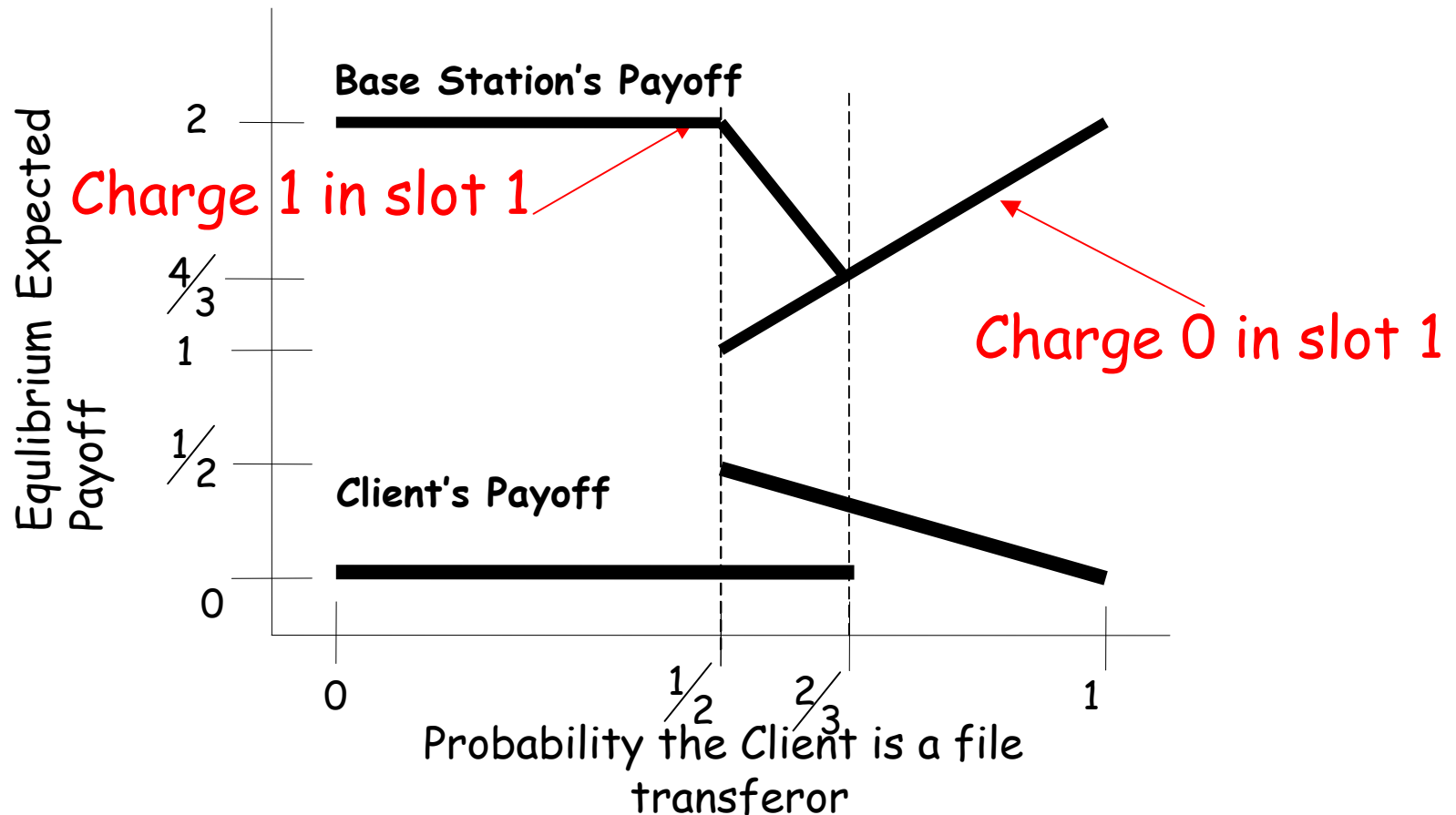
Bayesian Game: Fixed Length

Theorem 4 : The following are the Perfect Bayesian Equilibria.

Case	Slot 1 price	Slot 2 price	Client Strategy
$0 \leq p < 1/2$	1	1	Myopic
$1/2 \leq p \leq 2/3$ (2 PBE)	1	1	Mixed
	0	2	Pessimistic
$2/3 < p \leq 1$	0	2	Pessimistic

Bayesian Game: Fixed Length

Expected Payoffs



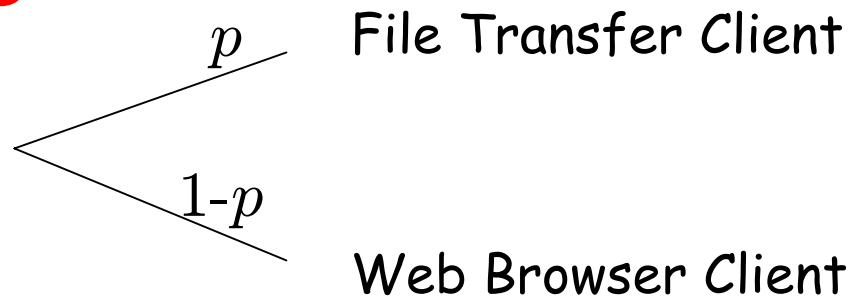
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Bayesian Game: Unbounded Length

Theorem 5

■ Assume



- $p > 0$
- K distributed on $\{1, 2, \dots\}$
- $P(K = i \mid K \geq i) > \delta$ for some $\delta > 0$
- $E[K \mid K \geq i]$ finite
- (Example: geometric)

Bayesian Game: Unbounded Length

Theorem 5 (cont'd)

■ Then ...

- Constant price is not a Perfect Bayesian Equilibrium.

■ Proof...

- Similar to Unbounded Length File Transfer proof.

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Ongoing Work

■ Model Variations

- Clients can make repeat visits
- Multi-Hop
 - A client acts as a someone else's base station (3 players)
- Dynamic Arrivals and Capacity Constraints